As the Crow Flies

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Purpose

The distance formula is often presented as a "rule" for students to memorize. This task is designed to help students develop an understanding of the meaning of the formula. It would be appropriate as an introduction (or review) of the distance formula for students who are familiar with the Pythagorean theorem and coordinate systems.

Task Overview

Compute the distance between two locations in a city with the streets laid out on an evenly spaced square grid. Then define a coordinate system and think about how to compute the distance by using the coordinates. *An activity sheet that gives students the complete task is included.*

Focus on Reasoning and Sense Making

Reasoning Habits

Focus in High School Mathematics: Reasoning and Sense Making

Analyzing a problem—seeking relationships; applying previously learned concepts; looking for hidden structure

Seeking and using connections across different representations

Reflecting on a solution—generalizing a solution

Process Standards

Principles and Standards for School Mathematics

Problem Solving—build new mathematical knowledge through problem solving

Connections—understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Representation—use representations to model and interpret physical, social, and mathematical phenomena

Standards for Mathematical Practice

Common Core State Standards for Mathematics

2. Reason abstractly and quantitatively.

4. Model with mathematics.

7. Look for and make sense of structure.

Focus on Mathematical Content

Key Elements

Focus in High School Mathematics: Reasoning and Sense Making

Reasoning with geometry—construction and evaluation of geometric arguments; multiple geometric approaches

Standards for Mathematical Content

Common Core State Standards for Mathematics

G-GPE-4. Use coordinates to prove simple geometric theorems algebraically.

G-GPE-7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Materials and Technology

As the Crow Flies activity sheet

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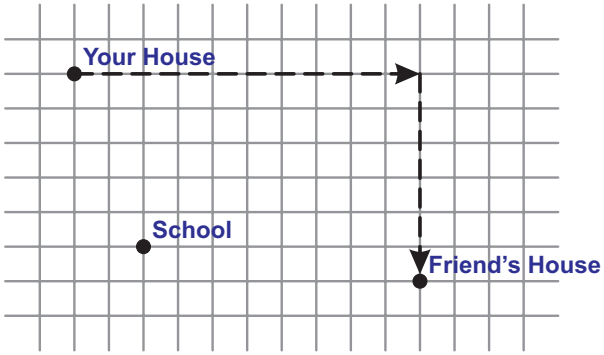
Use in the Classroom

You might begin by presenting the situation described on the activity sheet to be sure that your students understand the context. Students who live in rural areas might not be familiar with streets laid out in a grid. Be sure that your students understand that in the context, a *block* is a unit of length. Take care not to make any reference to the distance formula at this point.

Students should make sense of the problem for themselves. Use of a real-world context can help them make connections.

You might then ask the students to find the driving distance from their house to their friend's house, as described in question 1 on the activity sheet. You could have them work either individually or in groups on this question. Afterward, in a whole-class discussion, gather a range of student ideas about how to find this distance. As different students present their solutions, ask, "Why do you think that will work? Could you draw a diagram that will make this clearer?" Although encouraging a variety of solutions is helpful, you should also be sure to show a "simple" path, such as that shown in the representation below.

Students should express the reasoning that lies behind their answers as they solve the problem.



Next, ask your students to find the distance that they would travel if they could fly by helicopter—the "straight line" distance often described as distance "as the crow flies," discussed in question 2 on the activity sheet. As before, students could work either individually or in groups. If necessary, you might ask the students questions such as, "What will the direct path look like on your grid? If we include the path taken when driving, do you see a familiar shape? Do we know a theorem that might be helpful?" In the subsequent whole-class discussion, be sure that students see the usefulness of the Pythagorean theorem in finding the direct path.

Connecting the current situation to previous knowledge of the Pythagorean theorem provides a necessary foundation for understanding the context. Note the "hidden structure" in the problem—a right triangle.

Once the students see the connection to the Pythagorean theorem, you might ask them to "coordinatize" the situation, as described in questions 3 and 4 on the activity sheet. Again, you might have the students work individually or in pairs on these two items.

Coordinates are a useful means of modeling real-world problems.

Following a whole-class discussion of how coordinates might be useful, guide your students in developing a generalization of the situation, which should result in the emergence of the distance formula. Ask questions such as, "What distances did we find to help us calculate the direct distance? How could we determine the horizontal and vertical distance if we were given only the coordinates? How would we use these distances to find the length of the direct path?"

Connecting the differences in the coordinates of the points to the lengths of the sides of the right triangle is an important step in understanding the formula.

You might then ask the students to do an additional example with the ordered pairs (2, –5) and (–3, –7) to be sure that they understand the general pattern. You could then pose the general case of the distance between (x_1, y_1) and (x_2, y_2) to lead to the distance formula.

Seeing that the same reasoning can be repeated with different coordinates leads to a generalized formula to determine any distance.

Use in the Classroom—*Continued*

To wrap up the discussion, ask questions such as, "Does anyone know what this little fact is called? What might be a good name for this handy little fact? Where might you find this useful?"

Question 5 on the activity sheet poses a final problem, which you might use to help your students practice and extend their understanding of the use of coordinates in work with distance. Note that students may interpret *distance* in this question to mean either "distance on streets," as in question 1, or "distance as the crow flies," as in question 4. You should prompt them to include the latter interpretation if they do not arrive at it on their own.

Focus on Student Thinking

Students should recognize that the Pythagorean theorem is useful in finding the direct distance between two locations, since the horizontal and vertical distances, along with the direct path, form a right triangle. To help them make this connection, you might ask, "Can we form a familiar shape by considering the horizontal and vertical distances? How do we know that a right angle is formed?"

Because they are working on a grid, the students should recognize that they could easily use coordinates. To help with this process, you might ask, "What does this picture remind you of? Where would be a logical location for the origin? What coordinates would the other two locations have?" If necessary, explain, "The house is two blocks to the left and five blocks up, so its coordinates would be (–2, 5). And the coordinates for your friend's house would be (8, –1)."

To help your students see that they can find the horizontal distance by subtracting the x-coordinates and the vertical distance by subtracting the y-coordinates, you might ask, "How could you find the vertical and horizontal distances without counting?" The students should see that the Pythagorean theorem would then be useful in finding the direct distance.

To help your students move toward the general formula, you might ask them to compute the distances between other pairs of points without drawing a picture. You could then pose the problem of finding the distance d between (x_1, y_1) and (x_2, y_2). Encourage your students to follow the reasoning that they used with particular points to note that $|x_2 - x_1|$ is the horizontal distance and $|y_2 - y_1|$ is the vertical distance. Then they could apply the Pythagorean theorem as before: $(|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 = d^2$. Students might also recognize that since they are squaring, they can drop the absolute value. Solving for d then produces the distance formula, which in this case will be the positive solution.

Assessment

For homework, you might ask students to write a clear explanation of the distance formula and why it works. You could also use question 5 to offer your students additional practice in using the distance formula.

Resources

Common Core State Standards Initiative (CCSSI). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math).* Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010. http://www.corestandards.org.

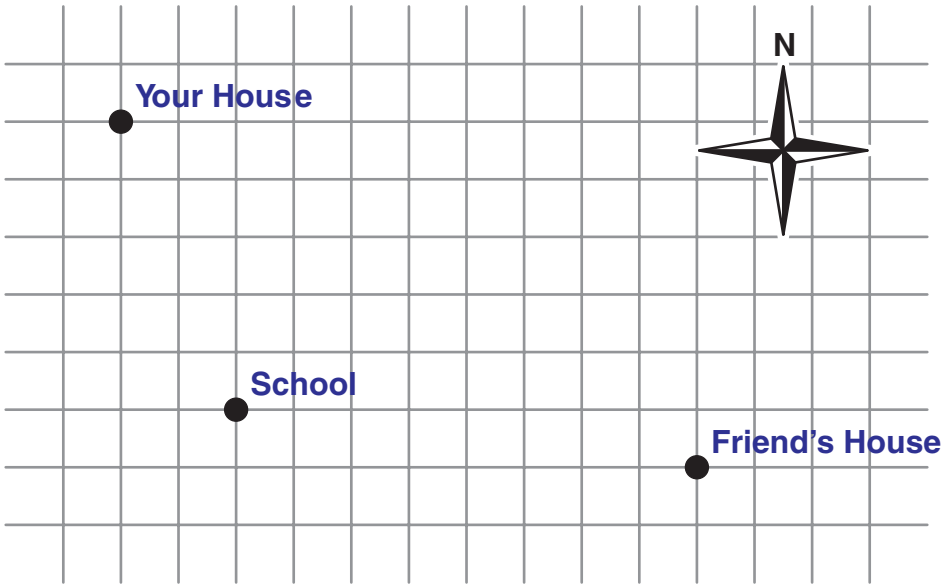
National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

————. *Focus in High School Mathematics: Reasoning and Sense Making.* Reston, Va.: NCTM, 2009. Example 13, pp. 52–53.

As the Crow Flies

Student Activity Sheet

Suppose that the city in which you live has a system of evenly spaced perpendicular streets, forming square city blocks. The map below shows your school; your house, which is located two blocks west and five blocks north of the school; and your best friend's house, which is located eight blocks east and one block south of the school.



1. How many blocks would you have to drive to get from your house to your friend's house? Draw a path that you would drive, and calculate the distance.

2. What if you could use a helicopter to fly straight from your house to your friend's house? Draw the path that you would take. How could you find the distance "as the crow flies"?

3. Establish a coordinate-axis system, using the school as the origin. What would the coordinates be for your house? For your friend's house?

As the Crow Flies

Student Activity Sheet (Continued)

4. How could you use the coordinates to calculate the distance "as the crow flies" from your house to your friend's house?

5. Suppose that your uncle lives two blocks east and one block south of the school and that you decide to stop by his house on the way home from your friend's house. Compute the round-trip distance from your house to your friend's house, to your uncle's house, and then back to your own house.

    a. Draw a picture to show your solution.

    b. Show how you could find the round-trip distance by using only coordinates.